The vertical curve—a baffling problem

TO THE EDITOR: In computing elevations on a vertical curve by addition of increments, in his article in the June 1955 issue (p. 60), Edmund R. Preston, Jr., made one error in arithmetic at Sta. 30 + 50 which makes all elevations beyond this station incorrect.

The vertical curve presents a baffling problem, mostly because of the way it is generally handled. The classic formula for a parabola with the axis vertical is $x^2 = 4fy$, where f = focal length for problems dealing with parallel rays. Since $4f$ is a constant, why not use it? Letting $4f = 1/k$, we can write $y = kx^2$. See Fig. 1. Working from the apex, this equation is simple.

Engineers generally know the simple-beam formula, $m = wL^2/8$. Some know that m at any point x is $\frac{wx}{2}(L - x)$. However, few realize that this is also $\frac{w}{2}x(L - x)$, which is also $kx(L - x)$. This last form also applies to ordinates for a vertical curve from a chord, as shown in Fig. 2.

In the example given by Mr. Preston, $a = +2$, $b = -3$, $L = 18$, and the elevation of the P.C. is 136.00 at Sta. 25 + 76. The elevations from this point are $136.00 + ax - kx^2$. From Fig. 1,

$$A = a - b = 2 + 3 = 5$$
$$k = A/2L = 5/36 = 1/7.2$$
$$Y = \frac{a^2 L}{2A} = 7.2 \text{ ft}$$
$$X = aL/A = 2(18)/5 = 7.2 \text{ stations}$$

or $a/2k = 7.2$ stations.

Since elevations are wanted at even stations and half stations, the second difference is found in Table I, letting I = increment = 0.5 stations.

Note that when $x = I$ or zero, kI^2 is half the normal difference.

$$2kI^2 = 1/14.4 = 0.06944$$
$$k(2xI + I^2 - x^2) = 0.0692$$

and $kx^2 = 0.0294$

STATIONS	dy	Σdy	ELEVATIONS
32 + 96 (apex)			143.2000
32 + 50	0.0294		143.1706
32 + 00	0.0692	0.0986	143.072
31 + 50	0.06944	0.16804	142.904
31 + 00	0.06944	0.23748	142.6665
30 + 50	0.06944	0.30692	142.3596
30 + 00	0.06944	0.37636	141.9832
29 + 50	0.06944	0.44580	141.5374
29 + 00	0.06944	0.51524	141.0222

Checking from the apex,

$$y = (3.96)^2/(7.2) = 2.178.$$

Therefore the elevation at Sta. 29 + 00 is (143.2000 − 2.178), or 141.022 exactly.

The equation for direct computations from the horizontal chord through the P.C. is shown in Fig. 2. The equation becomes $z = k\,x\,(C - x) + 136.00$. See Table II for computations.

From these computations it is easy to see that if differences are used they should be started at the apex. Direct computations from the horizontal tangent at the apex or from a horizontal chord through either the P.C. or P.T. will reduce the amount of work as well as the likelihood of errors.

The value of the constant k is a key to the sight distance, and when k = zero, the parabola is a straight line. As k increases or decreases the sight distance decreases. If the assumed eye height and object height are chosen, then the k value can be determined.

It seems to me that kx^2 or $kx(C - x)$ are so simple to use that trying to compute elevations by the summation of differences is a waste of time. Any error in the summation of differences carries over, as shown in Mr. Preston's example.

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FIG. 1

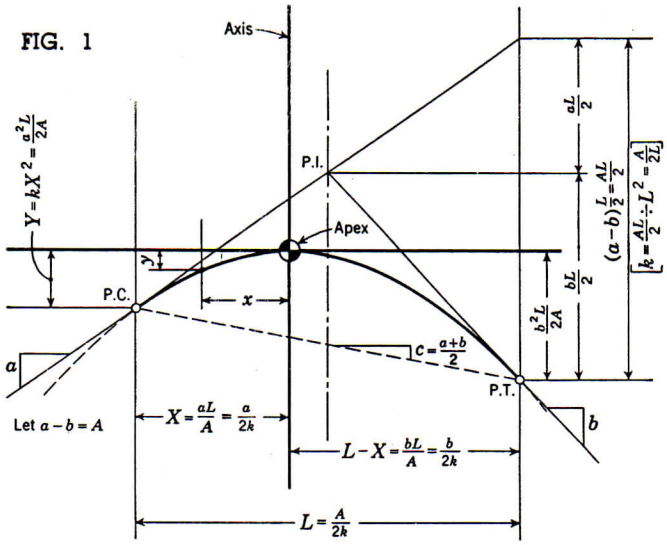


FIG. 2

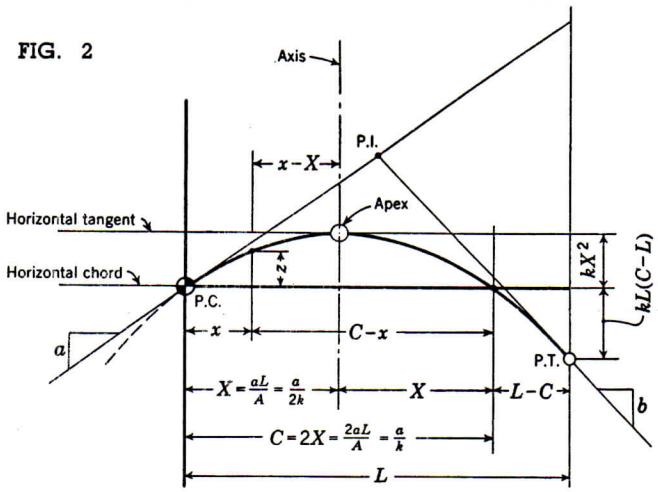


TABLE I. Computation of second difference

	FIRST DIFFERENCE	SECOND DIFFERENCE
$y_4 = k(x + 3I)^2 = k(x^2 + 6xI + 9I^2)$		
$y_3 = k(x + 2I)^2 = k(x^2 + 4xI + 4I^2)$	$k(2xI + 5I^2)$	
$y_2 = k(x + I)^2 = k(x^2 + 2xI + I^2)$	$k(2xI + 3I^2)$	$2kI^2$
$y_1 = k(x + 0)^2 = kx^2$	$k(2xI + I^2)$	$k(2xI + I^2 - x^2)$
	kx^2	

TABLE II. Solutions for equation of Fig. 2

STATION	x	$C - x$	$x(C - x)$	$1/7.2\ z$	ELEVATIONS
25 + 76	0	14.4	0	0	136.000
27	1.24	13.16	16.3184	2.266	138.266
29	3.24	11.16	36.1584	5.022	141.022*
31	5.24	9.16	47.9984	6.666	142.666
33	7.24	7.16	51.8384	7.1998	143.1998
35	9.24	5.16	47.6784	6.622	142.622
37	11.24	3.16	35.5184	4.933	140.933
39	13.24	1.16	15.3584	2.133	138.133
41	15.24	− 0.84	−12.8016	− 1.778	134.222
43	17.24	− 2.84	−48.9616	− 6.801	129.199
43 + 76	18.00	− 3.60	−64.8000	− 9.000	127.000

* The starred elevation checks with the above computations.

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